4.15

Properties of Logarithmic Functions and Graphs

Product Rule: $\log_a (MN) = \log_a M + \log_a N$

Quotient Rule: $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

Power Rule: $\log_a (M)^r = r \log_a M$

***Note**: If M = N then $\log_a M = \log_a N$, if $\log_a M = \log_a N$, then M = N,

and if $M^x = M^y$, then $x = y$.

See pg. 169 for examples.

<u>Example</u>: Proving the Product Rule for Logarithms

Prove $\log_b(RS) = \log_b R + \log_b S$.

Solution: Let $x = \log_b R$ *and* $y = \log_b S$. The corresponding statements are

$$b^x = R \ \ and \ \ b^y = S. \text{ So,}$$
$$\begin{aligned} \text{RS} &= b^x \cdot b^y \\ &= b^{x+y} && \text{first prop. of exponents} \\ \log_b(RS) &= x + y && \text{change to logarithmic function} \\ &= \log_b R + \log_b S && \text{def. of x and y} \end{aligned}$$

<u>Expanding the Logarithm of a Product</u>

Example:

Assume *x* and *y* are positive, use properties of logarithms to write $\log(27x^2y^5)$ as a sum of logarithms or multiples of logarithms.

$$
\begin{aligned}
\log(27x^2y^5) &= \log 27 + \log x^2 + \log y^5 && \textit{product rule} \\
&= \log 3^3 + \log x^2 + \log y^5 && 27 = 3^3 \\
&= 3\log 3 + 2\log x + 5\log y && \textit{power rule}
\end{aligned}
$$

Expanding the Logarithm of a Quotient

Example:

Assuming *x* is positive, use properties of logarithms to write $\ln\left(\frac{\sqrt{x^4+2}}{x}\right)$ as a sum or difference of logarithms or multiples of logarithms.

$$
\begin{aligned}
\ln\left(\frac{\sqrt{x^4+2}}{x}\right) &= \ln\frac{\left(x^4+2\right)^{\frac{1}{2}}}{x} \\
&= \ln\left(x^4+2\right)^{\frac{1}{2}} - \ln x && \textit{quotient rule} \\
&= \frac{1}{2}\ln\left(x^4+2\right) - \ln x && \textit{power rule}
\end{aligned}
$$

Condensing a Logarithmic Expression

Example:

Assuming *x* and *y* are positive, write $\ln x^2 - 3\ln(xy)$ as a single logarithm.

$$
\begin{aligned}
\ln x^2 - 3\ln(xy) &= \ln x^2 - \ln(xy)^3 && \textit{power rule} \\
&= \ln x^2 - \ln\left(x^3y^3\right) \\
&= \ln\frac{x^2}{x^3y^3} && \textit{quotient rule} \\
&= \ln\frac{1}{xy^3}
\end{aligned}
$$

Change of Base Formula for Logarithms

For positive real numbers *a*, *b* and *x* with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b} \quad \text{OR} \quad \log_a x = \frac{\log\ x}{\log\ a} \quad \text{OR} \quad \log_a x = \frac{\ln x}{\ln a}$$

<u>Evaluating Logarithms by Changing the Base</u>

Example:

$$a)\ \log_3 18 = \frac{\ln 18}{\ln 3} = 2.6309... \approx 2.63$$

$$b)\ \log_4 20 = \frac{\log 20}{\log 4} = 2.1609... \approx 2.16$$

Logarithmic Functions with *b* > 1, *f(x)* = log$_b$ *x*

Domain $(0,\infty)$

Range: All reals

x-intercept = (1 ,0)

y-intercepts: none

Increasing on its domain

No symmetry: neither even nor odd

No local extrema

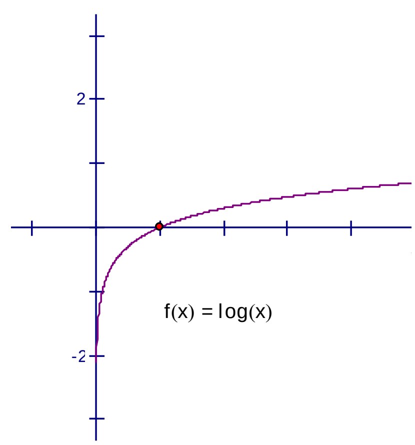
No horizontal asymptotes

Vertical asymptote: $x = 0$

End behavior: $\lim_{x\to\infty} \log_b x = \infty$

Passes through (1,0) and (b,1)

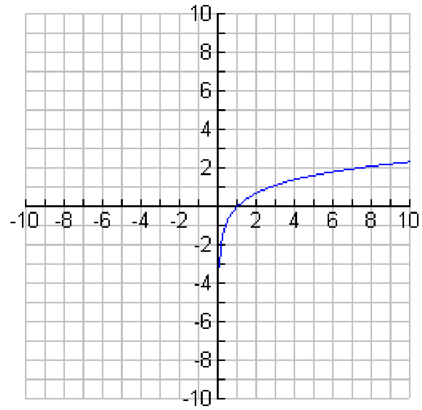
One-to-one



The Natural Logarithmic Function

$f(x) = \ln x$

Domain: $(0, \infty)$



Range: All reals

Continuous on $(0, \infty)$

Increasing on $(0, \infty)$

No symmetry

Not bounded above or below

No local extrema

No horizontal asymptotes

Vertical asymptote: $x = 0$

End Behavior: $\lim_{x \to \infty} \ln x = \infty$

(Do examples of graphing.)